Instability and convection
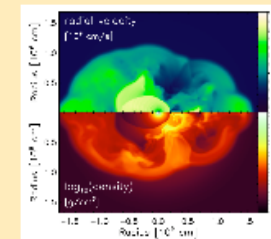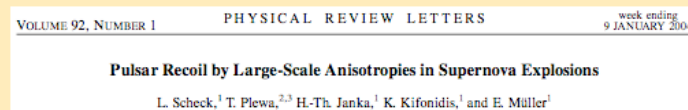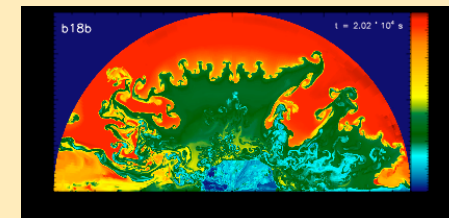# in core collapse supernovae

Thierry Foglizzo
CEA Saclay

Collaborators: Pascal Galletti, Tatsuya Yamasaki (CEA Saclay)
Thomas Janka, Leonhard Scheck (MPA Garching)

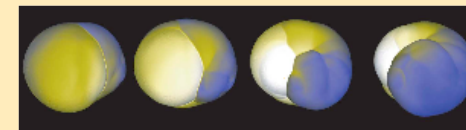# *Some beautiful (possible) consequences of SASI*

-Neutron star kicks *(Scheck et al. 2004, 2006)*

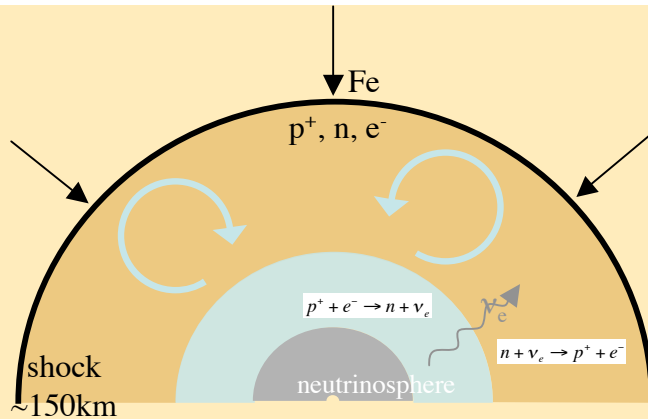-Seed the H/He mixing in the neutrino-driven explosion of 1987A, $1s<t<10^4$s *(Kifonidis et al. 2006)*

-Spin up of the neutron star *(Blondin & Mezzacappa 2007)*
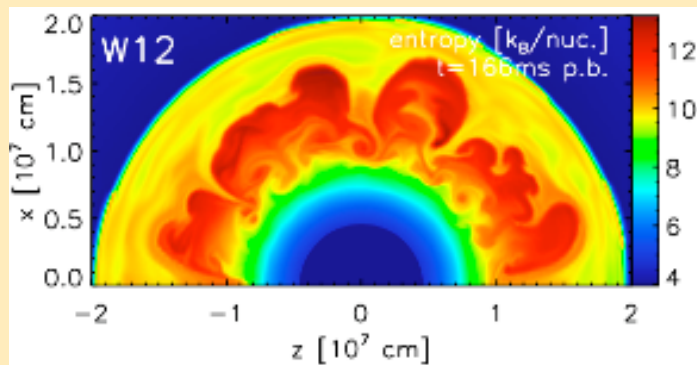
- New explosion mechanism driven by acoustic waves, initiated by the advective-acoustic cycle *(Burrows et al. 2005, 2006)*
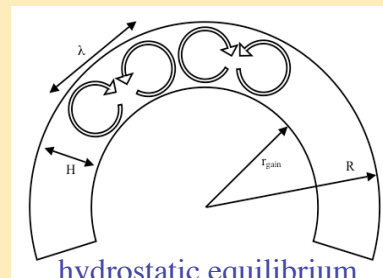
Fe

p^+, n, e^-

$p^+ + e^- \rightarrow n + \nu_e$

e

$n + \nu_e \rightarrow p^+ + e^-$

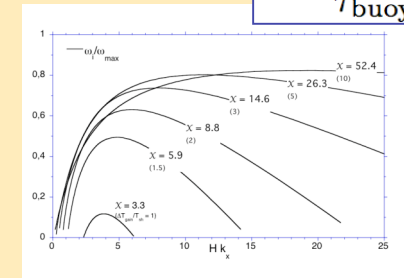shock ~150km

neutrinosphere

convection ? SASI 2003 ? SASI 2006 ?
advective-acoustic cycle ?

- Convection in the gain region, low l ?
(Herant, Benz & Colgate 1992)

$$\chi \equiv \frac{\tau_{adv}}{\tau_{buoy}} > 3$$

hydrostatic equilibrium
(Chandrasekhar 1961)

Foglizzo, Scheck & Janka (2006)

W12   entropy [k$_B$/nuc.]   t=166ms p.b.

W12F   entropy [k$_B$/nuc.]   t=166ms p.b.
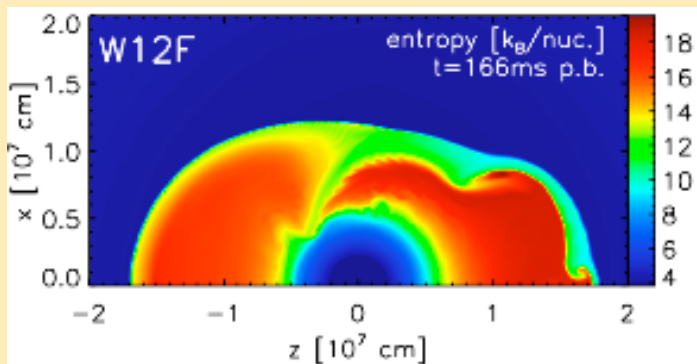
- l=1,2 SASI in an adiabatic flow:
advective-acoustic cycle ?
Blondin et al. '03, Ohnishi et al. '06, Foglizzo et al. '07, Yamasaki & Yamada '07
or
purely acoustic mechanism ??
Blondin & Mezzacappa '06, Blondin & Shaw '07

see also the advective-acoustic cycle in an adiabatic or isothermal accellerated flow
Foglizzo & Tagger '00, Foglizzo '01, '02, Foglizzo et al. '05

What do we understand of SASI ?

« Aero-acoustic » instabilities

- advected perturbations
- acoustic feedback

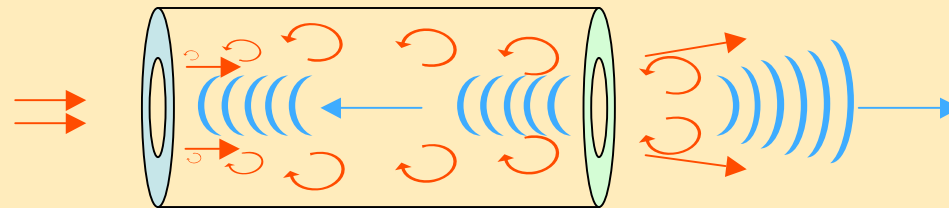• « vortical-acoustic » cycle

vibrations in Ariane 5:
segmented solid propergol
Mettenleiter, Haile & Candel (2000)
J. of Sound and Vibration 230, 761

impinging shear layers
Rockwell, D. (1983), AIAA J., 21, 645

whistling kettle
Chanaud & Powell (1965)
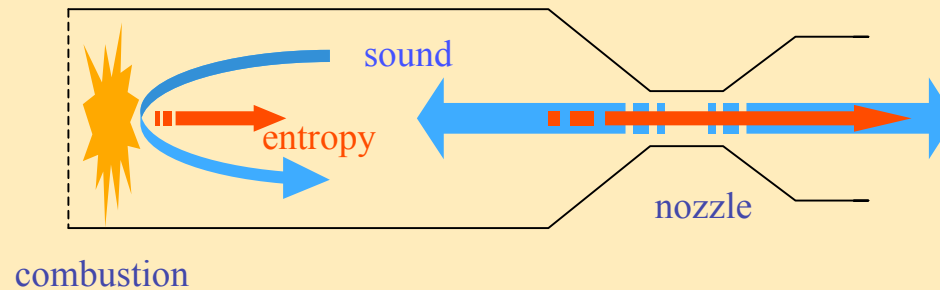*J. Acoust.Soc. Am. 37, 902*

• « entropic-acoustic » cycle

sound

entropy

nozzle
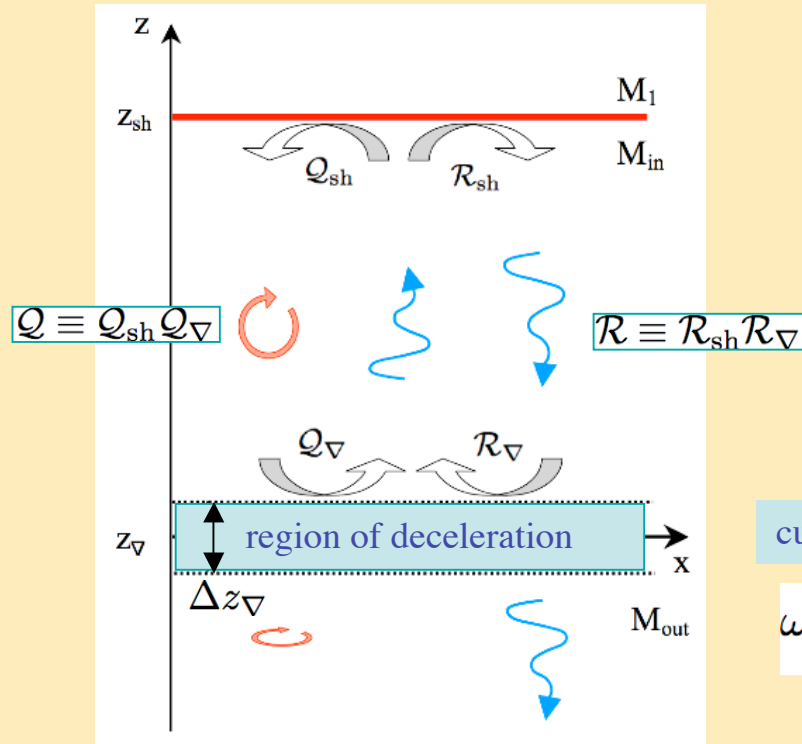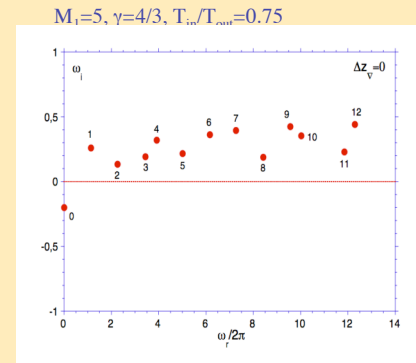
combustion

rumble instability of ramjet combustors
Abouseif, Keklak & Toong (1984)
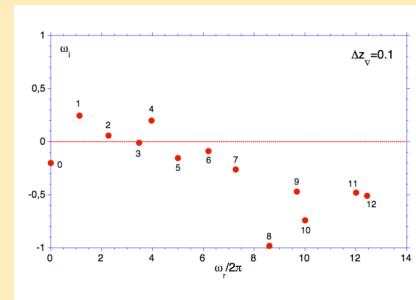*Combustion Science and Technology, 36, 83*

The simplest example of an advective-acoustic instability

-parallel adiabatic flow, localized coupling (γ, M_1, $\Delta\Phi$, Δz_∇)

-2-D perturbations $\omega, k_\perp$

z_{sh} M_1 M_{in}

$\mathcal{Q}_{sh}$ $\mathcal{R}_{sh}$

$\mathcal{Q} \equiv \mathcal{Q}_{sh}\mathcal{Q}_\nabla$ $\mathcal{R} \equiv \mathcal{R}_{sh}\mathcal{R}_\nabla$

$\mathcal{Q}_\nabla$ $\mathcal{R}_\nabla$

z_∇ region of deceleration x

Δz_∇ M_{out}

$M_1=5,\ \gamma=4/3,\ T_{in}/T_{out}=0.75$

$\Delta z_\nabla = 0$

$\Delta z_\nabla = 0.1$

cut-off frequency

$$\omega_\nabla \sim \frac{v_\nabla}{\Delta z_\nabla}$$

$$\tau_\mathcal{Q} \equiv \frac{1+\mu_{in}\mathcal{M}_{in}}{1-\mathcal{M}_{in}^2}\frac{z_{sh}-z_\nabla}{|v_{in}|}$$

$$\tau_\mathcal{R} \equiv \frac{2\mu_{in}}{1-\mathcal{M}_{in}^2}\frac{z_{sh}-z_\nabla}{c_{in}}$$

$$\mathcal{Q}e^{i\omega\tau_\mathcal{Q}} + \mathcal{R}e^{i\omega\tau_\mathcal{R}} = 1$$

$$\mathcal{Q}_c \equiv \frac{\frac{4\mu_{in}}{\mathcal{M}_{in}}(1-\mathcal{M}_{in}^2)\left(1-\frac{1}{\mathcal{M}_1^2}\right)}{\mu_{out}\frac{c_{in}^2}{c_{out}^2}+\mu_{in}\frac{\mathcal{M}_{out}}{\mathcal{M}_{in}}}\frac{\mathcal{M}_{out}+\mu_{out}}{1+\mu_{out}\mathcal{M}_{out}}\left[\frac{1-\frac{c_{in}^2}{c_{out}^2}+\frac{k_x^2 c_{in}^2}{\omega^2}(\mathcal{M}_{in}^2-\mathcal{M}_{out}^2)}{(\gamma+1)(1-\mu_{in}\mathcal{M}_{in})(\mu_{in}^2+2\mu_{in}\mathcal{M}_{in}+\mathcal{M}_1^{-2})}\right]$$

$$R = \frac{\mu_{out}M_{in}c_{in}^2-\mu_{in}M_{out}c_{out}^2}{\mu_{out}M_{in}c_{in}^2+\mu_{in}M_{out}c_{out}^2}\left(\frac{1+\mu_{in}M_{in}}{1-\mu_{in}M_{in}}\right)\frac{\mu_{in}^2-2\mu_{in}M_{in}+\frac{1}{M_1^2}}{\mu_{in}^2+2\mu_{in}M_{in}+\frac{1}{M_1^2}}$$

$$\mu^2 \equiv 1-\frac{k_x^2 c^2}{\omega^2}(1-\mathcal{M}^2)$$

$$|\mathcal{R}| \leq 1$$
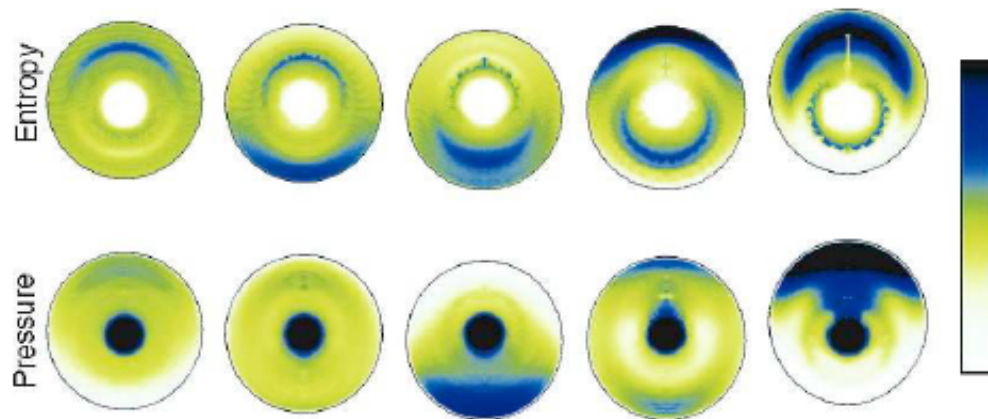
Asymmetry without convection:
numerical simulation of a stalled accretion shock
on a neutron star

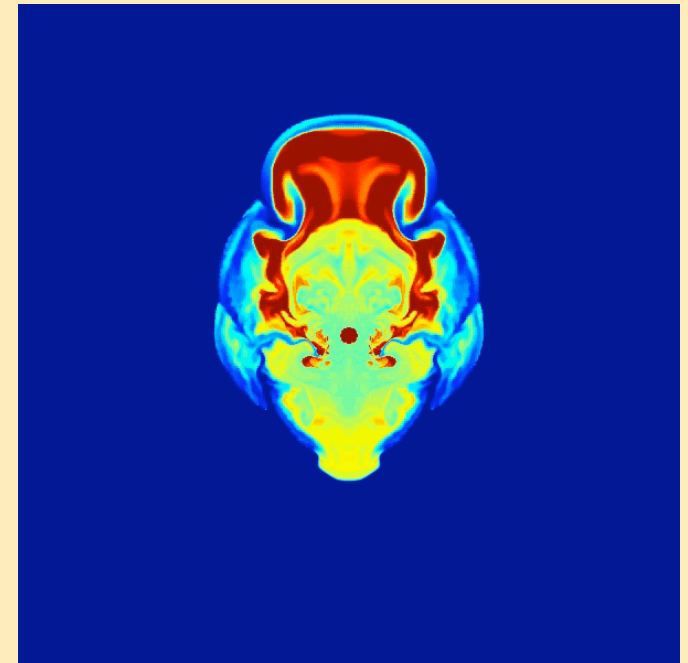**Evidence for a vortical-acoustic cycle** *(Blondin et al. 2003)*
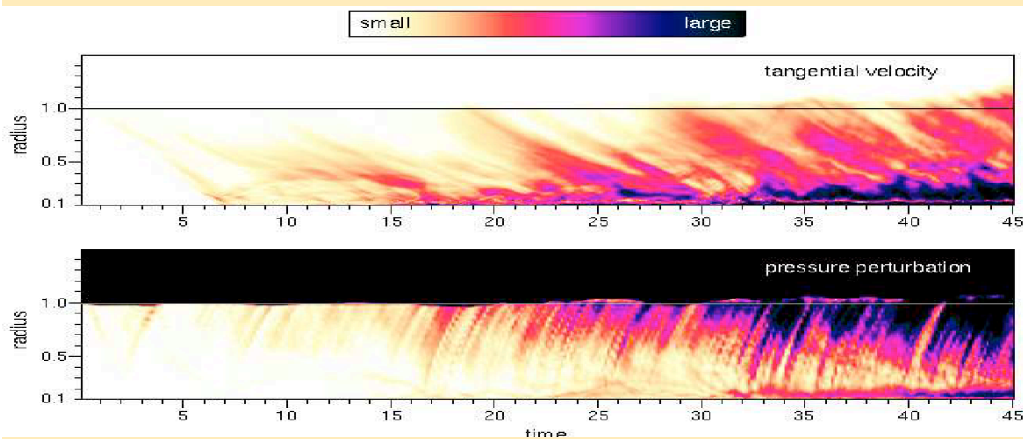
Toy model:
-perfect inviscid gas
$-\gamma=4/3$
-no heating
-simplified cooling $L \sim \rho^{\beta}T^{\alpha}$
-Euler equation
-adiabatic shock

A purely acoustic cycle ? *(Blondin & Mezzacappa 2006)*

What is the instability mechanism behind SASI ?

I- Linear stability analysis
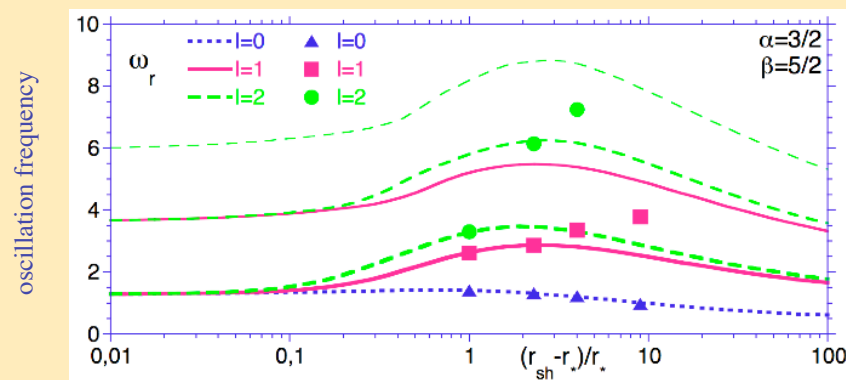
SASI toy model:
-perfect inviscid gas
$-\gamma=4/3$
-no heating
-Euler equation
-adiabatic shock
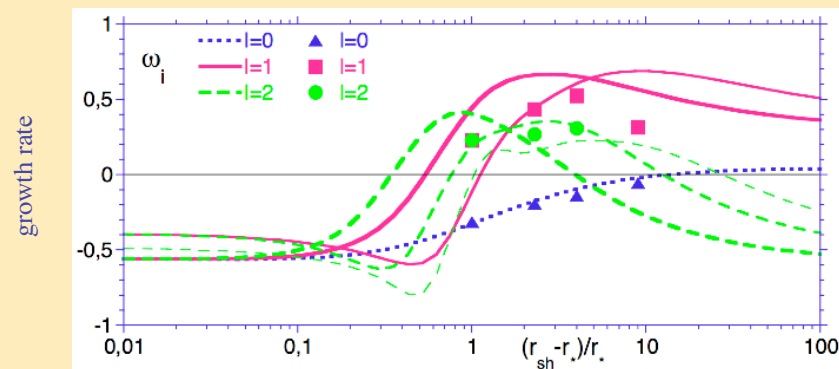
Simplified cooling function
$$L \sim \rho^\beta T^\alpha$$

$\alpha=6$, $\beta=1$ (settling flow)
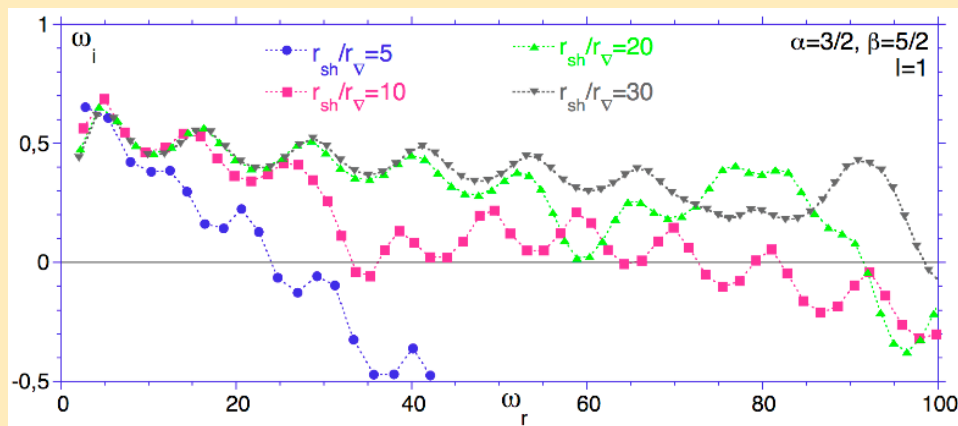$\alpha=3/2$, $\beta=5/2$ (cooling runaway)

2 timescales: -growth time $\omega_i^{-1} \sim \tau_{adv}$
-oscillation period $2\pi/\omega_r \sim \tau_{ac}$
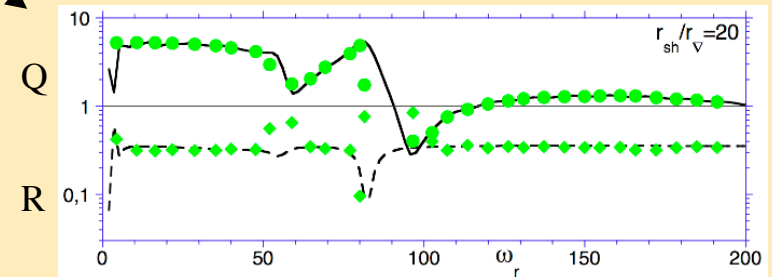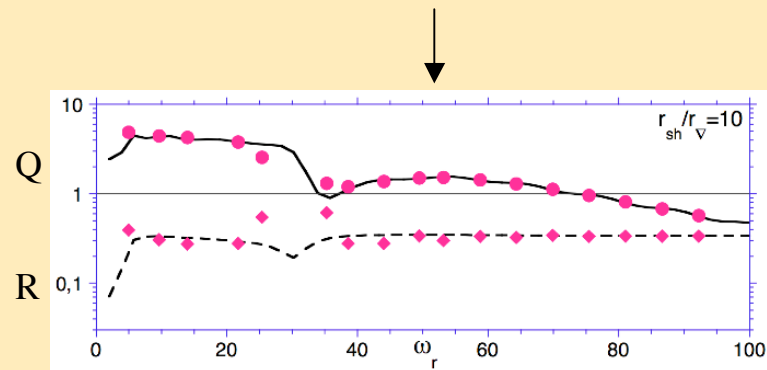
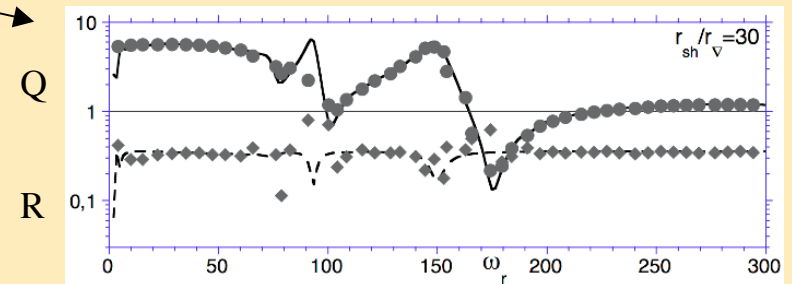# What is the instability mechanism behind SASI ?

(Foglizzo, Galletti, Scheck & Janka 2007)

II- Beyond the eigenfrequencies: efficiencies Q, R of 2 cycles

2 separate methods coincide:
- WKB approximation of $Q(\omega_r)$, $R(\omega_r)$
- direct extraction of Q, R from the
    eigenspectrum oscillations

- The advective-acoustic cycle is unstable: Q>1
- The purely acoustic cycle is stable: R<1

SASI is due to an advective-acoustic instability

Conclusions

Neutrino-driven convection in the gain region cannot be held responsible for pulsar kicks

SASI is due to the instability of an advective-acoustic cycle Q>1, R<1
      - growth time ~ advection time
      - oscillation period ~ acoustic time

Simple toy model of the advective-acoustic instability in a decelerated flow
      - sensitivity to the radial size Δz_∇ of the deceleration region
      - gradient cut-off ω_∇ -> low frequency, low l

Numerical simulations: advection of vorticity, grid size in the region of deceleration ?

Core-collapse consequences

Δz_∇

      - radial size of the cooling region of deceleration ?
      - rotation of the core: kick-spin alignment ? spin up ?